Written Exam for the M.Sc. in Economics Winter 2014-2015

# ADVANCED MACROECONOMETRICS

Final Exam

January 21, 10:00 – January 23, 10:00

PLEASE NOTE that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish. If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The paper must be uploaded as one PDF document (including the standard cover and the appendices). The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Absalon.

FOCUS ON EXAM CHEATING: In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

# Practical Information

Note the following formal requirements:

- This is an *individual* examination. You are not allowed to cooperate with other students or other people, see the *focus on exam cheating* above.
- The assignment consists of Sections 1-5 with 19 questions to be answered. *Please answer all questions.*
- The exam paper should not exceed 20 pages. A maximum of 20 pages of supporting material (graphs, estimation output, etc.) can accompany the paper as appendices. You may refer to the computer output in the appendices when answering the questions. Also, you may add clarifying comments in the output as part of your answer.
- All pages must be numbered consecutively and marked with your *exam number*. You should *not* write your name on the exam paper.
- *Your paper must be uploaded on the course page in Absalon at the given time.* The exam paper (including supporting material) must be in *PDF-format* and collected in *one file only*; the uploaded file must be named `1234.pdf`, where `1234` is your exam number.

The purpose of the examination is to assess your understanding of the cointegrated VAR (CVAR) model, your ability to use statistical procedures to make inference on the equilibrium structures and the dynamic adjustment properties, as well as your ability to interpret the results. Most questions in the examination are applied, concerning the empirical example outlined below. When you answer these empirical questions, please explain and motivate your answer as detailed as possible, preferably with reference to the underlying theory.

Regarding the data for the exam paper, please note the following:

- All assignments are based on *different* data sets. You should use the data set located in the Excel file `Data1234.xls`, where `1234` is your exam number.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

# 1 Background and Statistical Model

This project examination seeks to estimate an IS-LM model for a certain country C using the cointegrated vector autoregressive model. Country C is a relatively large and closed economy. Before 1990, the country had its own currency but in January 1990 it joined a monetary union with 14 other member countries. After joining the union, regulations regarding international capital flows were liberalized.

A simple linear version of the well-known IS-LM model stipulates that the output gap is determined by the *ex post* real interest rate in an equilibrium IS-curve,

$$y_t - \gamma_0 t = \gamma_1 \left( R_{bt} - \pi_t \right), \tag{1.1}$$

where $y_t$ is the log of output, $t$ is a linear trend variable used to construct a simple measure of the output gap, $R_{bt}$ is the long term bond rate, while $\pi_t$ is the inflation rate. In addition, the LM curve suggests that the equilibrium money demand is determined by output and the opportunity cost of holding money,

$$m_t = \gamma_2 y_t + \gamma_3 \left( R_{bt} - R_{mt} \right), \tag{1.2}$$

where $m_t$ is the money stock, and $R_{mt}$ is the short-term interest rate capturing the interest rate on money holdings such that $R_{bt} - R_{mt}$ is the opportunity cost of holding money.

The data consists of the five variables

| | | |
|---|---|---|
| $m$ | : | Real money stock (nominal M3 divided by the output deflator) |
| $y$ | : | Real output (GDP) |
| $\pi$ | : | Inflation (Change in output deflator in percent *p.a.*) |
| $R_m$ | : | Short interest rate (deposit rate in percent *p.a.*) |
| $R_b$ | : | Long interest rate (10 year bond rate in percent *p.a.*) |

All variables are observed quarterly from 1975 : 1 to 2012 : 4. For the analysis below, define the $p = 5$ dimensional vector of variables, $x_t = (m_t, y_t, \pi_t, R_{mt}, R_{bt})'$.

[1] Assume that all variables in $x_t$ are I(1) and that the IS-LM model in (1.1) and (1.2) is a good description of economy C. If you performed an analysis with the cointegrated VAR, what would you expect to find in terms of cointegration relationships.
State the Granger representation for the cointegrated VAR for this particular case, and explain how it could be used to discuss the short-run and long-run impact of shocks to the system.

[2] Now assume that (1.1) and (1.2) do hold, but that inflation, $\pi_t$, behaves as a stationary process over the considered sample. Explain what you would now expect to find in terms of cointegrating relationships. Also discuss the implications for the Granger representation.

[3] Finally, assume that all variables in $x_t$ are in fact I(1), but that $m_t - y_t$, $R_{bt} - R_{mt}$, and $R_{bt} - \pi_t$, behave as stationary processes. Explain what you would now expect

to find in terms of cointegrating relationships and the Granger representation. How would that relate to the IS-LM model?

[4] Set up and estimate an empirically relevant VAR($k$) model for the data in $x_t$,

$$x_t = \sum_{i=1}^{k} \Pi_i x_{t-i} + \phi D_t + \epsilon_t, \tag{1.3}$$

for $t = 1, 2, ..., T$, $\epsilon_t$ independently and identically distributed $N(0, \Omega)$, initial values, $x_{-k+1}, ..., x_{-1}, x_0$, fixed, and where the vector $D_t$ contains potential deterministic variables, such as a constant, a trend, and dummy variables relevant for the empirical analysis. Carefully explain the steps you take and motivate the choices you make. State the assumptions for the model, and test that the model is well specified.
In practice it may not be possible to find a model that is acceptable in all directions, just do as well as you can.

# 2 Estimation and Cointegration Rank

[5] Write the log-likelihood function for the unrestricted VAR model in (1.3) as a function of $\Omega$ and $\theta = \{\Pi_1, ..., \Pi_k, \phi\}$. Use that the maximum likelihood estimator for $\Omega$, given the parameters in $\theta$, can be found as $\hat{\Omega}(\theta) = T^{-1} \sum_{t=1}^{T} \epsilon_t(\theta)\epsilon_t(\theta)'$, to show that the concentrated likelihood function takes the form

$$\log L(\theta) = c - \frac{T}{2} \log \left| \hat{\Omega}(\theta) \right|,$$

where $c$ is a constant that does not depend on $\theta$.

[6] Derive the error correction form of the VAR model in (1.3) with the lag-length of the empirical model above.
Write the characteristic polynomial for the model and explain how the presence of unit roots is related to the reduced rank of a certain parameter matrix in the error correction form of the model.

[7] A friend of yours has found the following statement in a discussion on an internet blog: "*Most applications using cointegrated VAR models are rubbish, because in most cases it doesn't make sense to assume that all variables in the system have unit roots*".
Write your reaction as an answer to the internet blog.

[8] Determine the cointegration rank, $r$ say, in your preferred model for $x_t$ using all available information. Explain, in particular, how to calculate the likelihood ratio statistic and how to simulate the relevant asymptotic distribution for the case of your preferred model.

# 3 Hypotheses Testing

[9] Impose the reduced rank as determined above, $\Pi = \alpha\beta'$.
Test if the interest rate spread, $R_{bt} - R_{mt}$, and the real interest rate, $R_{bt} - \pi_t$, behave as stationary variables around the included deterministic variables in your model, cf. question 3. Explain how to formulate the hypothesis and how to calculate the degrees of freedom.
Next, test if inflation, $\pi_t$, is stationary around the included deterministic variables in your model, cf. question 2.
Also test if $R_{bt} - R_{mt}$, $R_{bt} - \pi_t$, or $\pi_t$ are stationary without the deterministic components.

[10] Test the hypothesis that one of the stochastic trends, $CT_{1t} = \sum_{i=1}^{t} u_i$ say, affects only money and income and in the same magnitude, i.e. corresponding to a Granger representation of the form

$$\begin{pmatrix} m_t \\ y_t \\ \pi_t \\ R_{mt} \\ R_{bt} \end{pmatrix} = \begin{pmatrix} 1 & b_{1,2} & \cdots & b_{1,(p-r)} \\ 1 & b_{2,2} & \cdots & b_{2,(p-r)} \\ 0 & b_{3,2} & \cdots & b_{3,(p-r)} \\ 0 & b_{4,2} & \cdots & b_{4,(p-r)} \\ 0 & b_{5,2} & \cdots & b_{5,(p-r)} \end{pmatrix} \begin{pmatrix} CT_{1t} \\ CT_{2t} \\ \vdots \\ CT_{(p-r)t} \end{pmatrix} + Y_t + A_t,$$

where $Y_t$ is a stationary process, and $A_t$ is a function of initial values and potential deterministic variables. Explain how to formulate the hypothesis and how to calculate the degrees of freedom.

[11] Now test the hypothesis that one of the stochastic trends is determined by cumulated shocks to income, i.e. $CT_{1t} = \sum_{i=1}^{t} \epsilon_{y,i}$. Explain how to formulate the hypothesis and how to calculate the degrees of freedom.
Also test whether one of the common trends is determined by cumulated shocks to the interest rate spread, $CT_{1t} = \sum_{i=1}^{t}(\epsilon_{R_m,i} - \epsilon_{R_b,i})$.

[12] Test the hypothesis that each of the chocks in $\epsilon_t$ has only transitory effects on the variables in $x_t$.
How many shocks with only transitory effects can you at most have in your preferred system?

# 4 Identification

Now we want to consider a restricted cointegration space,

$$\beta^c = (\beta_1^c, \beta_2^c, ..., \beta_r^c) = (H_1\varphi_1, H_2\varphi_2, ..., H_r\varphi_r),$$

where $H_j$ is a known matrix and $\varphi_j$ is a vector with parameters to be estimated, $j = 1, 2, ..., r$.

[13] Explain when a set of restrictions identify the cointegrating relationships, and how the condition for identification can be checked.

[14] Identify the long-run relationships in your empirical model. Explain the steps you take and the choices you make.

Give an economic interpretation of the long-run relationships and the equilibrium adjustment and relate to the IS-LM model.

[15] Consider the Granger representation for the preferred model and carefully interpret the results.

[16] Now imagine that the short interest rate $R_{mt}$ is under control of the central bank of the country. In particular, assume that the expected–or rule-based–monetary policy is given by the estimated equation for $\Delta R_{mt}$, while

$$\epsilon_{R_m,t} = \Delta R_{mt} - E\left(\Delta R_{mt} \mid x_{t-1}, ..., x_{t-k}\right),$$

measures unexpected monetary policy shocks.

A theoretical economist suggests the following definition: "Inflation is *controllable* by the central bank in the long run, if unexpected monetary policy shocks affect the inflation rate in the long-run".

Explain how you could test the null hypothesis that inflation is not controllable by the central bank in the long run, and perform a Wald test for this hypothesis in your empirical model.

Explain why this hypothesis is more difficult to test using a likelihood ratio test.

# 5 Extensions

[17] (Great Moderation) Since 1990, many countries have experienced smaller variation of many economic variables (at least up to the financial crisis). This has sometimes been called the *great moderation*. Assume that this can be modelled by a change in the covariance structure, such that the VAR($k$) model becomes,

$$x_t = \sum_{i=1}^{k} \Pi_i x_{t-i} + \phi D_t + \epsilon_t, \quad t = 1, 2, ..., T,$$

with $x_{-k+1}, ..., x_{-1}, x_0$ fixed, and

$$\epsilon_t \sim \begin{cases} N(0, \Omega_1) & \text{if} \quad t < 1989:4 \\ N(0, \Omega_2) & \text{if} \quad t \geq 1990:1, \end{cases}$$

where $\Omega_2$ implies smaller variances of individual variables than $\Omega_1$. Modify the likelihood function from question 5 to the new heteroskedastic case. State the maximum likelihood estimators for $\Omega_1$ and $\Omega_2$ given the other parameters, $\theta = \{\Pi_1, ..., \Pi_k, \phi\}$, $\hat{\Omega}_1(\theta)$ and $\hat{\Omega}_2(\theta)$ say. Do you think in this case, that you get closed form estimators for $\theta$?

In this heteroskedastic model, how would you perform and interpret an impulse-response analysis?

[18] (INFERENCE ON CONTEMPORANEOUS CAUSAL STRUCTURES) Consider a four dimensional system of estimated residuals from a VAR model, $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \epsilon_{4t})'$. The table below reports correlations and conditional correlations between residuals, with $p-$values for the hypotheses of zero correlations in brackets. Use the information to derive the class of observationally equivalent causal structures.

| | | | |
|---|---|---|---|
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{2t})$ | $=$ | $0.0543$ | [0.22] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{3t})$ | $=$ | $0.3557$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{3t} \mid \epsilon_{2t})$ | $=$ | $0.3626$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{3t} \mid \epsilon_{4t})$ | $=$ | $0.3000$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{3t} \mid \epsilon_{2t}, \epsilon_{4t})$ | $=$ | $0.3124$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{4t})$ | $=$ | $0.2015$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{4t} \mid \epsilon_{2t})$ | $=$ | $0.1949$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{4t} \mid \epsilon_{3t})$ | $=$ | $0.0216$ | [0.63] |
| $\mathrm{Corr}(\epsilon_{2t}, \epsilon_{3t})$ | $=$ | $0.3777$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{2t}, \epsilon_{3t} \mid \epsilon_{1t})$ | $=$ | $0.3840$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{2t}, \epsilon_{3t} \mid \epsilon_{4t})$ | $=$ | $0.3281$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{2t}, \epsilon_{3t} \mid \epsilon_{1t}, \epsilon_{4t})$ | $=$ | $0.3393$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{2t}, \epsilon_{4t})$ | $=$ | $0.1979$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{2t}, \epsilon_{4t} \mid \epsilon_{1t})$ | $=$ | $0.1912$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{2t}, \epsilon_{4t} \mid \epsilon_{3t})$ | $=$ | $0.0046$ | [0.92] |
| $\mathrm{Corr}(\epsilon_{3t}, \epsilon_{4t})$ | $=$ | $0.5186$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{3t}, \epsilon_{4t} \mid \epsilon_{1t})$ | $=$ | $0.4882$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{3t}, \epsilon_{4t} \mid \epsilon_{2t})$ | $=$ | $0.4890$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{3t}, \epsilon_{4t} \mid \epsilon_{1t}, \epsilon_{2t})$ | $=$ | $0.4577$ | [0.00] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{2t} \mid \epsilon_{3t})$ | $=$ | $-0.1925$ | [0.01] |
| $\mathrm{Corr}(\epsilon_{1t}, \epsilon_{4t} \mid \epsilon_{3t})$ | $=$ | $0.0213$ | [0.63] |
| $\mathrm{Corr}(\epsilon_{2t}, \epsilon_{4t} \mid \epsilon_{3t})$ | $=$ | $0.0026$ | [0.95] |

How can you use this information if you want to perform an impulse-response analysis using the estimated four-dimensional VAR model?

[19] (MEASUREMENT ERRORS) Consider a time series $x_t$ as generated from a VAR(1) with cointegration rank $r$,

$$\Delta x_t = \alpha\beta' x_{t-1} + \epsilon_t, \quad t = 1, 2, ..., T,$$

with $\epsilon_t$ independently and identically distributed, $N(0, \Omega)$. Now assume that you do *not* observe the actual variables in $x_t$ but only observe

$$y_t = x_t + w_t,$$

where the *measurement error,* $w_t$, is independently and identically distributed, $N(0, \Sigma_w)$. Derive an equation to show the behavior of the observed process, $y_t$. Explain why the variables in $y_t$ still cointegrate with the same cointegration rank, $r$, and the same cointegration vectors, $\beta$.
Would the same thing hold if the measurement error was an I(1) process, e.g.

$$w_t = w_{t-1} + \xi_t, \quad t = 1, 2, ..., T,$$

with $w_0 = 0$ and $\xi_t$ independently and identically distributed, $N(0, \Sigma_\xi)$?